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Wavelet-based semblance analysis applied to geophysical borehole data

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ABSTRACT

Wavelet-based semblance analysis is a relatively new measure which allows for the similarity investigation of local phase relationships between two data sets as a function of frequency (or wavenumber) and time (or distance). The potential for applying this method to geophysical datasets is explored, with quantitative results for two cases. The first case is where two types of data come from the same section of a borehole, and the second case where data comes from similar stratigraphic locations but from different boreholes. Source data comprises density and magnetic susceptibility data from the Bellevue (BV-1) and Moordkopje (MO-1) boreholes located within the Northern Lobe of the Bushveld Complex.

Key words: Wavelet, semblance analysis, correlation, Bellevue, Moordkopje

INTRODUCTION

The Bushveld Complex (here defined to refer only to the ultramafic-mafic layered rocks), is north of Pretoria in South Africa and host to 70% of the world's PGEs, comprises four limbs: the Western (including Far Western), Eastern, Northern (or Potgietersrus), and the sub-surface Southern (or Bethal) Limbs (Cawthorn and Walraven, 1998) (see Figure 5 in Appendix). Layering is present in vertical scales from mm's to hundreds of m's. It is laterally very continuous, and the overall stratigraphy of the lobes is remarkably similar (Eales and Cawthorn, 1996; Winter, 2001, p.222).

With the exception of several reversals and discontinuities (Ashwal et al., 2005), stratigraphic sections through the Eastern and Western lobes show upwardly decreasing trends in Mg# and mean An content and an upwardly increasing trend in Fe-enrichment, indicative of crystal fractionation (see Figure 6 in Appendix).

While layering of cumulus minerals may be easily explainable when consistent trends in density or falling liquidus temperature is present, the repetition of lithological layers cannot be explained in terms of a single process. Repetition of layers results in cyclicity within geophysical parameters, which may then be mathematically investigated. As an example, the minerals plagioclase and magnetite have densities 2.61-2.76 and 5.1-5.2 g/cm^3, and give rise to highly variable rock densities in the corresponding anorthosite and magnetitite layers in the Upper Zone. A similar result occurs with the magnetically strong magnetite and weak plagioclase. Comparison of these cyclicities would provide insight into lateral variations within the Bushveld Complex

While Fourier analysis is a tool used to determine the characteristic frequency/ies of a dataset (which remain constant throughout the dataset), and so provide valuable information about its cyclicity, wavelet analysis is able to monitor trends within each characteristic frequency throughout the length of the dataset. Similarly, Fourier-based semblance filtering is used to measure the degree of similarity between two datasets as a function of time or distance. Cooper and Cowan (2008) showed that the principles of wavelet analysis may be extended to this technique, with the advantage that changes in frequency content with time (or distance) may be analysed. This technique is termed wavelet-based semblance filtering. This technique has been successfully applied to synthetic sinusoidal data (Cooper and Cowan, 2008).

Application of wavelet-based semblance analysis by the comparison of real geophysical borehole datasets is performed here.

METHOD AND RESULTS

The Continuous Wavelet Transform

The Continuous Wavelet Transform (CWT),

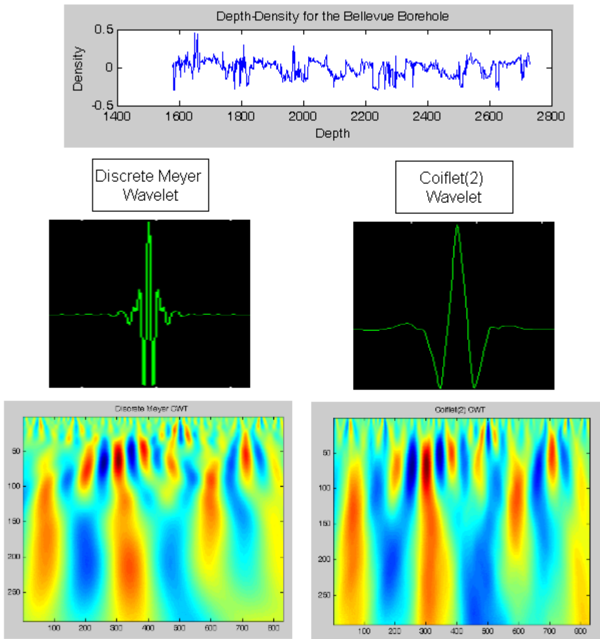
$$Wf(u,s) = \int_{-\infty}^{\infty} f(t) \frac{1}{|s|^p} \Psi^* \left(\frac{t-u}{s} \right) dt \qquad (1)$$

(Mallet, 1999), inputs data, f(t) and wavelet,

$$\Psi_s(t) = \frac{1}{|s|^p} \Psi \left(\frac{t}{s} \right) \qquad (2)$$

(which always integrates to zero). The parameter **u** allows the wavelet to be translated throughout the data, and the parameter **s** allows the wavelet to be stretched to various scales. The term p>0 compensates for horizontal stretching with vertical compression. The resulting CWT, *Wf(u,s),* is plotted using two axes as input parameters, and a colour scale for magnitude.

Figure 1 shows examples of wavelets, and the corresponding CWT-plot that results when applied to borehole density data.



Would it be possible to increase the fontsize of the axis. It is difficult to read. Figure 1. Continuous Wavelet Transform performed in Matlab on depth-density data from the Main Zone of the Bushveld Complex (Northern Lobe, Bellevue borehole), using two wavelets, the Discrete Meyer and Coiflet(2). Dark red indicates large positive values, and dark blue indicates large negative values.

While the type of wavelet used makes a marked impact on the shapes within the CWT-plot, it is clear that both wavelets produce similar trends. The vertical axis gives a quantity proportional to wavelength (notice that smaller values are nearer the top, with bigger values near the bottom), and the horizontal axis, a quantity proportional to depth. Tracking the positions of central intensity of the upper lobes from left to right, we see that wavelength initially decreased to a value proportional to 50 m (approximately), followed by an increase to 100 m and then a decrease back to 50 m.
By using a variety of wavelets on a set of data, trends within the data can be identified and scrutinised for plausibility.

Wavelet-based semblance analysis

A complex wavelet is used to calculate the complex CWT for the two datasets concerned. Torrence and Compo (1998) defined the cross-wavelet transform as

$$CWT_{1,2} = CWT_1 \times CWT_2^* \qquad (3)$$

This complex quantity has a cross-wavelet power of

$$A = |\, CWT_{1,2}| \qquad (4)$$

and a phase angle

$$\theta = \tan^{-1}(\Im(CWT_{1,2})/\Re(CWT_{1,2})) \qquad (5)$$

where θ ranges from –π to π. The semblance

$$S = \cos^n(\theta) \qquad (6)$$

is then a measure of phase correlation (where n is odd). Values close to 1 indicate high positive correlation, 0 indicates no correlation and -1indicates high negative correlation.

Larger amplitudes within the dataset may be of interest, in which case we then define (explain what D is called and does)

$$D = A \times S = |\, CWT_{1,2}| \times \cos^n(\theta) \qquad (7)$$

This is sometimes required to reduce the noise level. The details of this procedure may be found in (Cooper, 2008).

In the first instance this procedure is applied to density and magnetic susceptibility data from the Main Zone lithologies in the Bellevue (BV-1) borehole. Data was initially interpolated to 1m intervals.

First the CWT of both datasets was calculated:

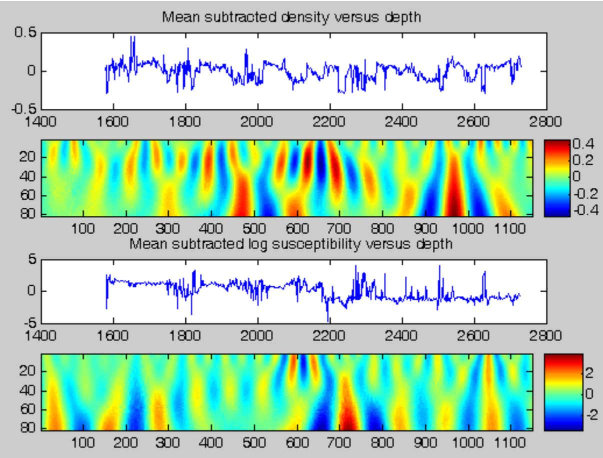


Figure 2. The complex Morlet(1-1) wavelet was used to determine the CWT of 2 geophysical properties within the same dataset. There is clear dissimilarity between the cyclicities within the two datasets as evidenced by the CWT(colourful)-plots.

The datasets appear generally to be out of sync with one another, with regards to phase and, more emphatically, amplitude.

The wavelet-based semblance analysis was then performed:

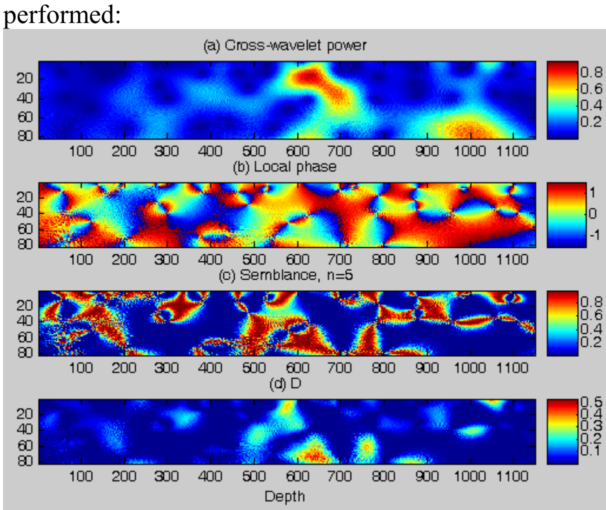


Figure 3. (a) Amplitude of the cross-wavelet transform. Note the predominance of values close to zero (dark blue). (b) Local phase angle of the cross-wavelet transform. Values close to zero (light green) imply phase correlation. (c) Semblance with high power to emphasise only strong phase correlations (indicated as values near 1). No individual cyclicity or cyclicity pattern is sustained throughout the entire dataset. (d) The final amplitude- and phase-dependent semblance shows low correlation throughout the entire dataset.

This serves as a good example of non-substantial correlation between datasets.

In the second case, wavelet-based semblance analysis was applied to the datasets of similar type but from different boreholes. Here density data from the Upper Zones of both the Moordkopje (MO-1) and Bellevue (BV-1) was considered. The Upper Zone of the MO-1 borehole extends for approximately 30 m before the Main Zone begins, so an equivalent portion was used from the BV-1 borehole. Care was taken to ensure that it had a similar stratigraphic location to the MO-1 dataset. Both boreholes are located in the Northern Lobe of the Bushveld Complex (see Figure).

The following results were obtained:

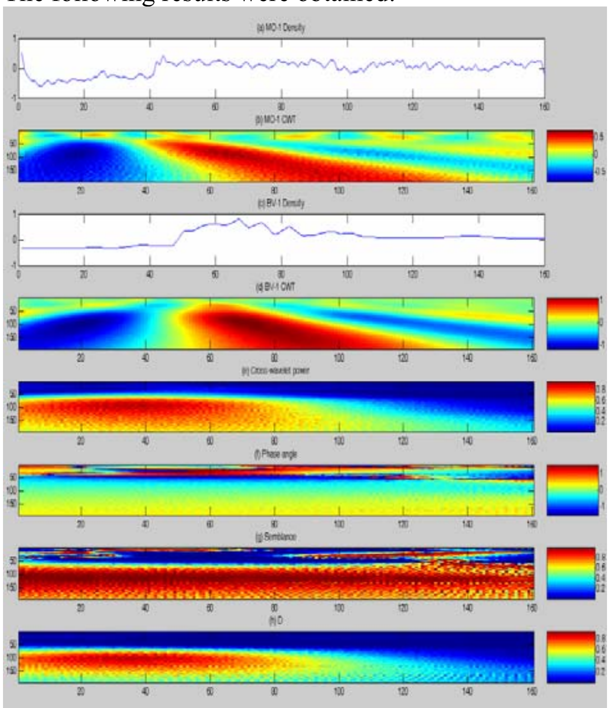


Figure 4. (a) MO-1 density data over depth of 32 m interpolated to 0.2m and (b) CWT of this data. (c) BV-1 density data over depth of 32 m interpolated to 0.2m and (d) CWT of this data. (e) Amplitude of cross-wavelet transform. Largest amplitudes are found within the first 100 data points (20 m) at scales greater than 75 (15 m wavelength) which correlates well to the input data. (f) Phase angle. Data shows strong positive phase correlation throughout the domain, particularly between scales of 100 and 150, also confirmed by (g) the Semblance. With very little noise, it is unnecessary to compute (h) D, in which amplitude overpowers phase correlation.

The Semblance in Figure 4(g) highlights a strong phase relationship between these two datasets at a scale of 110 (corresponding to 22 m). This is a pertinent example of high positive correlation between two datasets throughout the entire domain being studied.

CONCLUSIONS

Wavelet-based semblance analysis, which has previously only been applied to synthetic data, was successfully employed to determine phase relationships between real geophysical borehole datasets. Effects of interpolation, however, may need to be considered. This technique may be extended to other situations in Geophysics as well as other disciplines. In particular, by introducing relative scaling of datasets, it may be used to ascertain lateral variations in layer thicknesses within the various lobes of the Bushveld Complex. Monte Carlo sampling may become useful in determining the significance of correlations obtained.

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Appendix

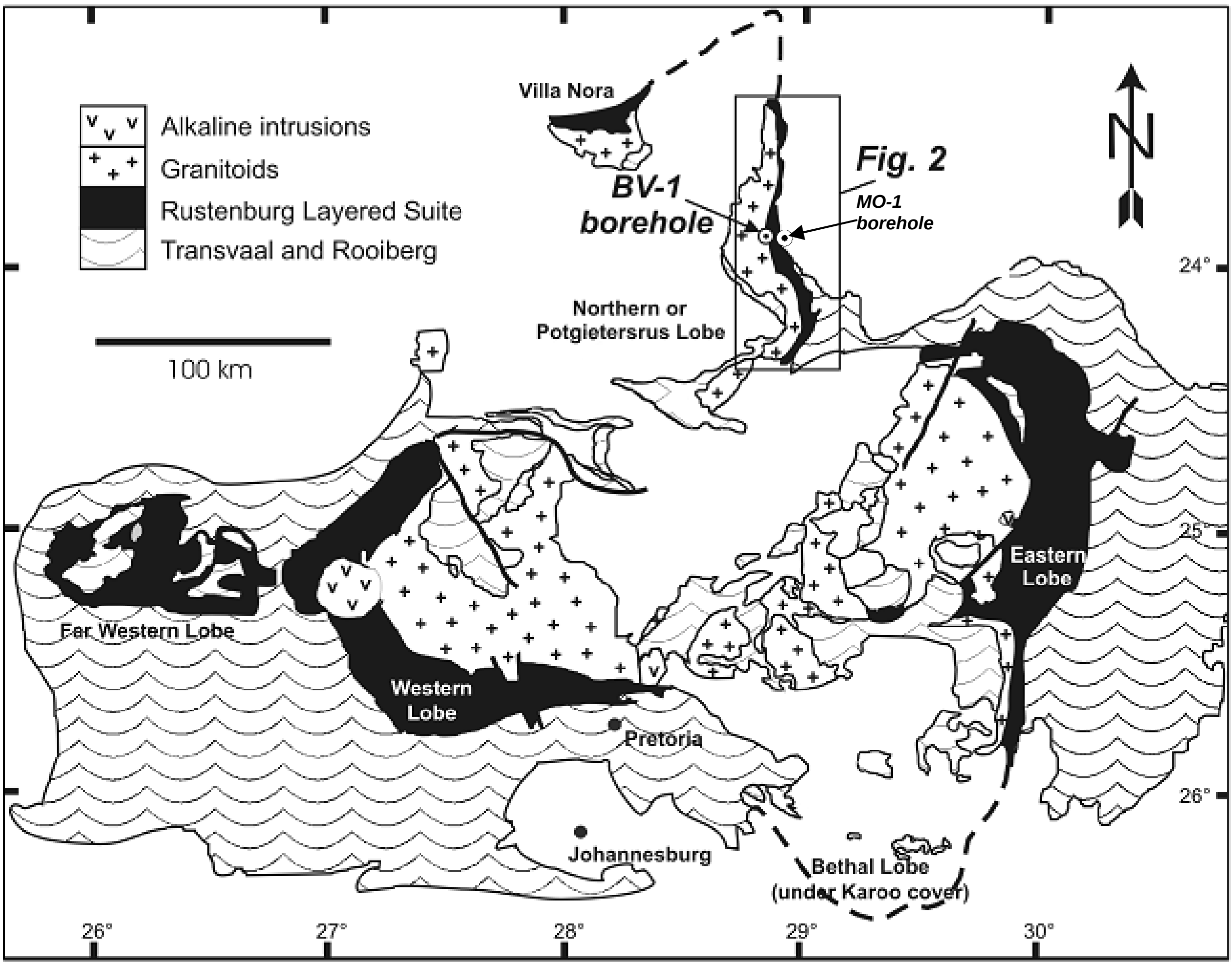

Figure 5. Map of the Bushveld Complex showing the localities of the Bellevue(BO-1) and Moordkopje(MO-1) drill holes (After (Roelofse et al., 2008; Ashwal et al., 2005)).

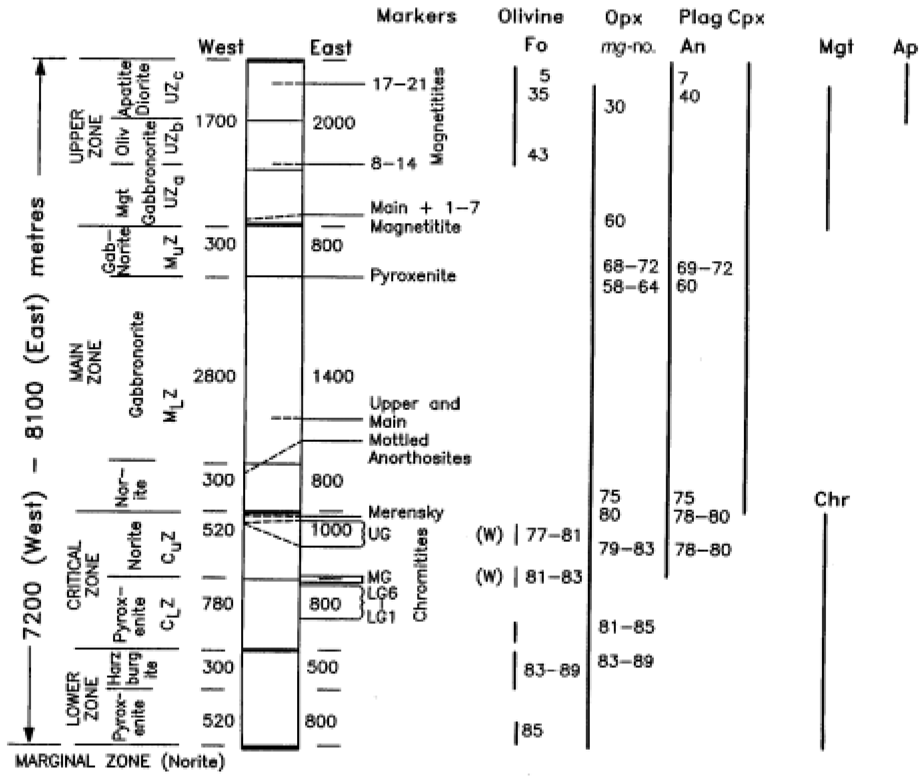

Figure 6. Generalised stratigraphic sections through the Eastern and Western Limbs of the Bushveld Complex indicating the Lower, Critical, Main and Upper Zones, their respective compositions, as well as vertical trends in Forsterite, Mg# and mean An content indicative of crystal fractionation (from Eales et al. 1996).