

Joint inversion of the magnetic anomaly due to a kimberlite pipe and its analytic signal

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ABSTRACT

A three-dimensional (3D) joint inversion of the magnetic anomaly due to a vertical right circular cylinder with arbitrary polarization and its analytic signal is presented. This geometry is commonly used for the modelling of kimberlite pipes. The analytical derivatives of the body parameters are derived and used in the inversion to avoid numerical problems that may result from the use of numerical derivatives based on finite differences. The inversion uses the Levenberg-Marquardt algorithm. It is found that the inversion of the analytic signal gives better results for the geometry of the source body than the inversion of the magnetic anomaly alone. Finally, the proposed joint inversion gives more stable results and is more robust than the separate inversions of these two data sets. The joint inversion also allows better estimation of the parameters of the cylinder than the inversion of the magnetic anomaly or its analytic signal alone. This is especially true for the orientation and strength of remanent magnetization when present.

Keywords: remanent magnetization, joint inversion, kimberlite, analytic signal, magnetic anomaly

INTRODUCTION

In the inversion of the magnetic anomaly of a vertical right circular cylinder with arbitrary polarization, a major problem is the presence of remanent magnetization (RM). This is especially the case in diamond exploration as the magnetization of many kimberlite pipes, which are a major source of diamonds, can be dominated by RM. The identification of RM has attracted much attention but many problems still remain. The inversion of magnetic data in the presence of RM is unstable; more specifically the RM direction and intensity are poorly resolved. The 2D analytic signal of the magnetic field can help solve this problem because it is independent of the inducing field direction and RM direction (Nabighian, 1972, 1974); however, the 3D analytic signal as defined by Roest *et al.* (1992) is not totally independent of the local geomagnetic field and RM orientations (Agarwal and Shaw, 1996; Li, 2006). A joint inversion of the magnetic anomaly and its analytic signal increases the stability of the inversion. To do this, we derive analytical expressions for the magnetic anomaly and its derivatives, as well as for its analytic signal. These analytical derivatives are used in all the inversions. The inversions are tested with synthetic and real data.

METHOD AND RESULTS

Derivation of the Analytical Derivatives

The analytic signal amplitude (hereafter simply referred to as the analytic signal) of the magnetic field is defined

as the amplitude of its total gradient. The derivation of analytical expressions for the x -, y - and z -derivatives of the magnetic anomaly is a prerequisite to the implementation of the analytic signal inversion and the joint inversion. The analytical expressions of the magnetic field of a right circular cylinder include two incomplete elliptic integrals and two complete elliptic integrals (Singh and Sabina, 1978). The analytical expressions for the derivatives of both the complete and incomplete elliptic integrals use Legendre's standard integrals. They are calculated by Carlson (1979) using the method of successive applications of the duplication theorem. Carlson and Notis (1981) present the algorithms and Chemam (2006) programmed them in Matlab language. These results are used in our calculations. The analytical derivatives are compared to derivatives calculated by forward finite differences to check the numerical results.

The model used to compare the numerical and analytical derivatives is a semi-infinite right vertical cylinder. The depth to the top of the cylinder is 99 m and its radius is 80 m. The total magnetization is 3.33 nT while the inclination and declination of the inducing magnetic field are 152° and -12°, respectively. A comparison of the analytical and finite difference derivatives is presented in Figure 1, (a) for the vertical derivatives, (b) for the x -derivatives, and (c) for the y -derivatives. From Figure 1 it is seen that there is an excellent agreement between the analytical and numerical derivatives.

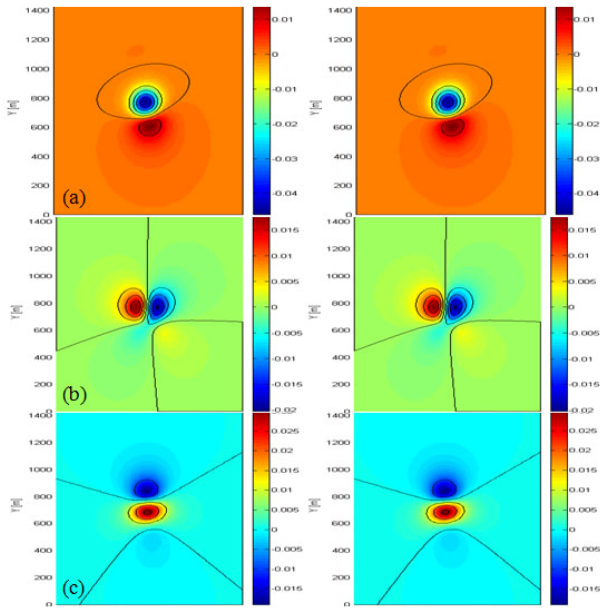


Figure 1. Analytical and numerical derivatives of the magnetic field of a right circular cylinder with regards to a) the x-, b) the y- and c) the z-direction.

Inversion of the total magnetic field anomaly

In the inversion of magnetic anomalies, it is generally assumed that the magnetization is strictly induced. Results are therefore erroneous when a strong remanent magnetization alters the total magnetization direction. To overcome this difficulty, Chemam (2006) implemented a 3D inversion of a vertical cylinder using the Levenberg-Marquardt algorithm including RM direction and strength as unknowns. The inversion proceeds in two steps: first step for the total magnetization and then, starting from those inversion results, a second step for remanent magnetization. The magnetic anomaly of a finite-extent cylinder is obtained from the difference between two semi-infinite cylinders. However, the results of the inversion are sometimes unstable and inaccurate.

In this work, the analytical expressions of the derivatives are used for computing the terms of the Jacobian matrix used in the inversion to increase the stability and accuracy of the inversion. Another improvement is that a new formula for the relationship between the induced magnetization and the total magnetization is substituted:

$$J_i^2 / J_{tot}^2 = 1/[1 + Q^2 + 2Q(a + b + c)] \tag{1}$$

where,

$$a = \sin I \sin I_r \tag{2}$$

$$b = \cos I \cos D \cos I_r \cos D_r \tag{3}$$

$$c = \cos I \sin D \cos I_r \sin D_r \tag{4}$$

In the above expressions, J_i is the induced magnetization intensity, J_{tot} is the total magnetization

intensity, Q is the Koenigsberger ratio, I is the inclination of the geomagnetic field, D is the declination of the geomagnetic field, and I_r and D_r are the inclination and declination of the remanent magnetization, respectively. J_{tot} is obtained from Chemam’s technique. Equation 1 allows for a simpler estimation of Q .

The algorithm for the inversion of the magnetic anomaly is tested on synthetic models and a real field case. Figure 2 presents an example of the application of the algorithm to a synthetic model. In this model the depth to the top of the cylinder is 60 m while the cylinder has a diameter of 160 m and a depth extent of 300 m. Both the inclination and declination of the RM is 30°. During the inversion, an initial value of 80° for the inclination of the RM is used to test the robustness to the change of initial values. Other initial values are determined by Chemam’s technique.

Compared to the algorithm of Chemam (2006), our algorithm is more stable and the solutions are more accurate. The inversion results obtained with Chemam’s algorithm are 59 m for the depth to the top of the cylinder, 162 m for its diameter, 307 m for its depth extent, and 13° and 33° for the inclination and declination of the RM, respectively. These results are less accurate than our results (shown in Figure 2).

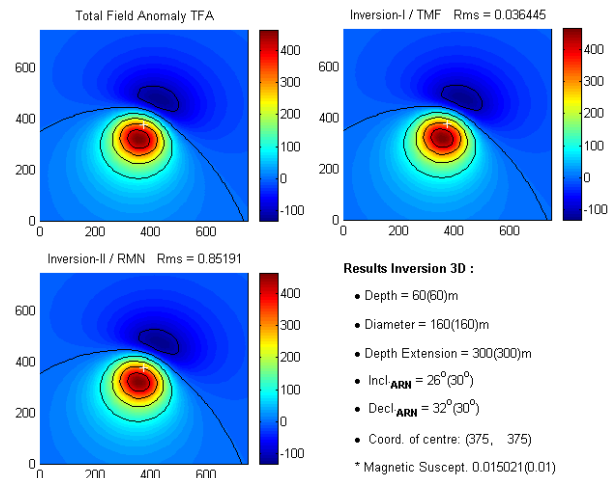


Figure 2. Results of the inversion of the total magnetic field anomaly. Geomagnetic field intensity = 57,170 nT, $I = 74^\circ$, $D = -12^\circ$, depth of the cylinder = 60 m, diameter of the model = 160 m, depth extent = 300 m, magnetic susceptibility = 0.01 SI. $I_r = 30^\circ$, $D_r = 30^\circ$, centre of the cylinder occurs at $x = y = 375$ m, Koenigsberger ratio Q is 5.

Inversion of the analytic signal

The 3D analytic signal is also called the amplitude of the total gradient and is given by:

$$g = \|\nabla F\| = \sqrt{(\partial F / \partial x)^2 + (\partial F / \partial y)^2 + (\partial F / \partial z)^2} \tag{5}$$

where F is total magnetic field anomaly and g is the analytic signal. The 3D analytic signal produced by the anomaly due to a vertical circular cylinder with arbitrary polarization has, in most cases, a symmetric shape (Agarwal and Shaw, 1996). When the total inclination is higher than 60° , the analytic signal is nearly independent of the total inclination and total declination, including the remanent magnetization and inducing field. Contrary to the magnetic field, the analytic signal has a weak dependence on the direction of the inducing magnetization and remanent magnetization.

The Levenberg-Marquardt algorithm is again used during inversion of the analytic signal. The algorithm for inversion of the analytic signal is tested using synthetic models and a real field example. Figure 3 presents the results of the inversion of the analytic signal of the previously used model (refer to Figure 2). An initial value of 80° is again used for the inclination of the RM. Other initial values are determined by Chemam's method. The results of the analytic signal inversion give a better estimate of the inclination of the remanent magnetization than the inversion of the magnetic field anomaly. Our inversion yields 60 m, 160 m, 323 m, 28° and 32° for depth to the top, diameter, depth extent, inclination and declination of the RM, respectively.

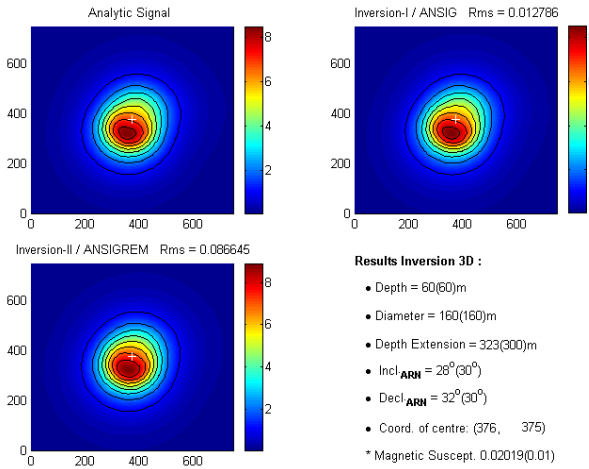


Figure 3. Results of the inversion of the analytic signal for the model of a right circular cylinder with depth, diameter, depth extent, RM inclination and declination of 30 m, 160 m, 300 m, 30° and 30° , respectively.

Joint inversion of the magnetic field and its analytic signal

Many authors (Vozoff and Jupp, 1975; Sasaki, 1989) have demonstrated the advantages of joint inversion using two types of data, related to the same property (resistivity in the case of the two papers cited above). A joint inversion provides better results than the separate inversion of each data set. Firstly, stability of the results is increased; secondly, the precision of the results is

enhanced. Similar improvements are expected for other types of data.

Formulation of the Joint Inversion

For the joint inversion problem, two inverse problems are involved:

$$F[m] + \varepsilon_1 = d_1^{obs} \quad (6)$$

$$g[m] + \varepsilon_2 = d_2^{obs} \quad (7)$$

where m is the vector of parameters. Linearizing the problem, we can write in matrix form:

$$A_1 \Delta m = \varepsilon_1 \quad (8)$$

$$A_2 \Delta m = \varepsilon_2 \quad (9)$$

where A_1 and A_2 are the Jacobian matrices for the magnetic field and its analytic signal and Δm is the matrix of the parameter changes. The Jacobian matrix of joint inversion is constructed as follows:

$$A = [N_1 W_1 A_1; N_2 W_2 A_2] \quad (10)$$

N_1 and N_2 are normalization factors for the two datasets and W_1 and W_2 are weighting factors. The error vector of the joint inversion is constructed as follows:

$$\varepsilon = [N_1 W_1 \varepsilon_1; N_2 W_2 \varepsilon_2] \quad (11)$$

The equation for the joint inversion problem is given by:

$$\varepsilon = A \Delta m \quad (12)$$

Starting from an initial value of m , the change to the parameter that yields a better fit of the calculated responses to the observed ones is given by:

$$\Delta m = (A^T A + \lambda I)^{-1} A^T \varepsilon \quad (13)$$

where λ is the damping factor.

Normalization

For each data set, the data are normalized as follows:

$$N = \sqrt{\frac{\sum_{i=1}^M d_i^2}{M}} \quad (14)$$

where d_i are data values and M is the number of data points.

Weighting of the two data sets

Weights W_1 and W_2 are obtained from the norms of F (magnetic field anomaly) and g (analytic signal anomaly):

$$W_1 = \frac{\|g\|}{\|F\| + \|g\|} \quad (15)$$

$$W_2 = \frac{\|F\|}{\|F\| + \|g\|} \quad (16)$$

Each data set therefore has an approximately equal influence on the parameter estimates.

Inversion of synthetic models

The algorithm of joint inversion is first tested on a synthetic model. Figure 4 presents the results of the joint inversion of the synthetic kimberlite model presented in Figure 2. An initial value of 80° for the inclination of the RM is used. Other initial values are evaluated by Chemam’s method.

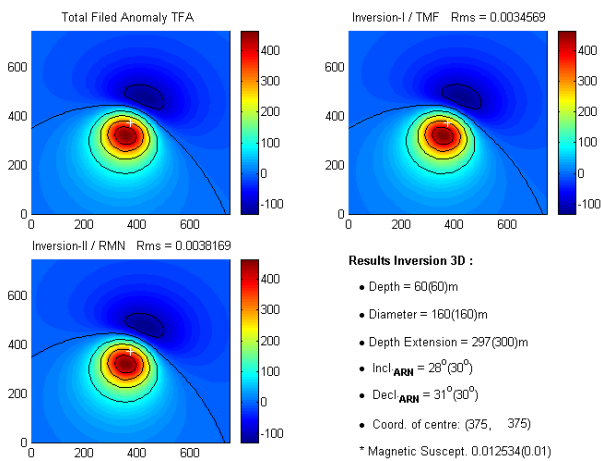


Figure 4. Calculated magnetic field from the results of the joint inversion of the kimberlite model of Figure 2.

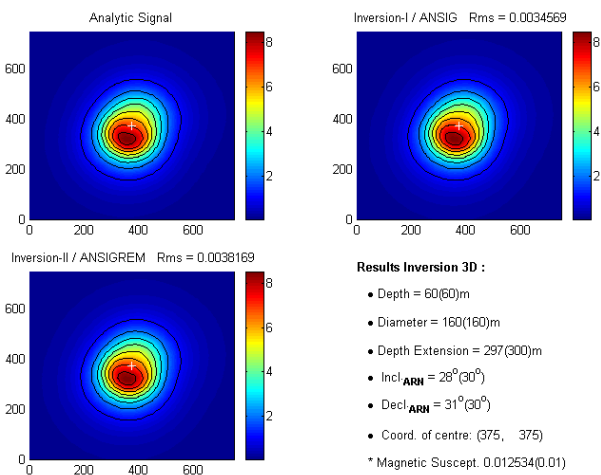


Figure 5. Calculated analytic signal from the results of the joint inversion of the model in Figure 2.

The results of joint inversion are 60 m for the depth to the top, 160 m for diameter, 297 m for the depth extent, 28° and 31° for the inclination and declination of RM, respectively. The interpreted depth extent is improved when compared with the results of the inversion of the analytic signal alone (refer to Figure 3).

Real data: the Peddie pipe

The algorithm is now applied to the Peddie pipe found in the town of Haileybury of Ontario, Canada. The Peddie pipe is very shallow. Its subcropping surface is covered by a thin sequence of glacial sediments, consisting of 1 to 3m of grey silty sand till overlain in places by up to 3m of glaciolacustrine clay. In order to reduce the influence of noise, data were upward continued to a height of 12 m. The results of the inversion of the magnetic anomaly are found to be 5 m for depth to the top, 43 m for the diameter, 94 m for the depth extent, 55° and -32° for the inclination and declination of RM, respectively (refer to Figure 6). These results should be compared to the results of the inversion on the analytic signal, namely 6 m for the depth to the top, 33 m for the diameter, 1 413 m for the depth extent, 28° and -17° for the inclination and declination of RM, respectively (not shown).

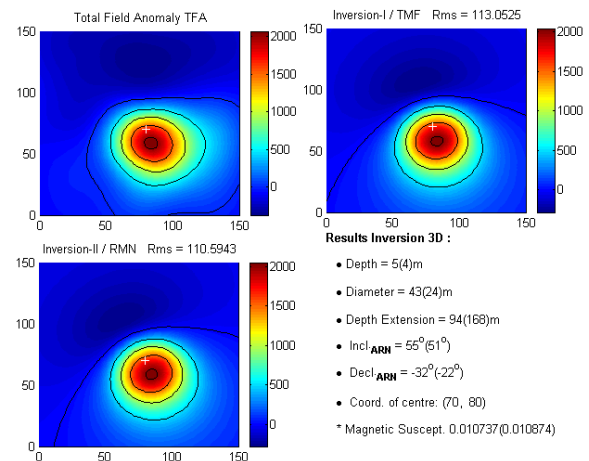


Figure 6. Calculated magnetic field from the results of the inversion of the magnetic anomaly of the Peddie pipe (upward continued to 12m).

The results of joint inversion presented in Figures 7 and 8 are 6 m for the depth to the top, 38 m for the diameter, 252 m for the depth extent, and 52° and -32° for the inclination and declination of RM, respectively. These results are more reliable than the results from the inversion of the magnetic field alone.

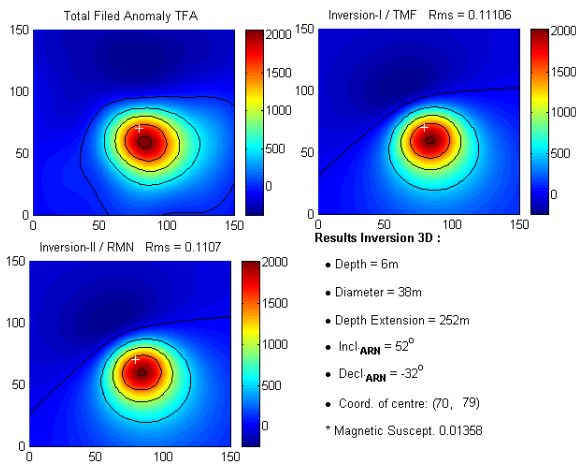


Figure 7. Calculated magnetic field from the results of the joint inversion of the Peddie pipe (upward continued to 12m).

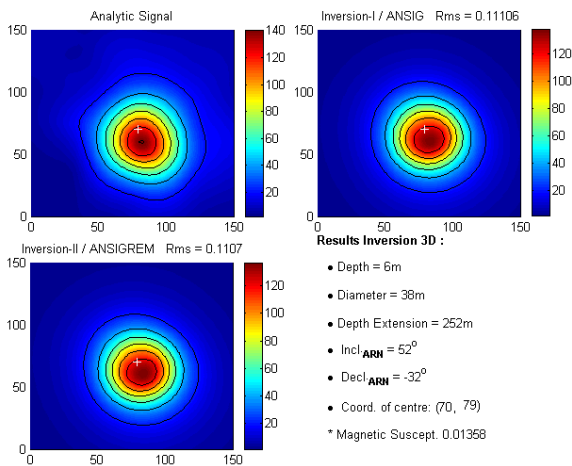


Figure 8. Calculated analytic signal from the results of the joint inversion of the Peddie pipe (upward continued to 12m).

CONCLUSIONS

The inversion of magnetic anomalies is improved by using analytical expressions in the Jacobian matrix and the substitution of a new formula for the relationship between the remanent magnetization and the induced magnetization. The analytic signal inversion is developed and in contrast to the inversion of the magnetic data, results are more stable and accurate. The joint inversion of the magnetic field and its analytic signal provides better results than the separate inversions of the magnetic anomaly or its analytic signal. The results of the joint inversion are more reliable and stable. This is especially true when the geomagnetic field inclination is high, or when the initial values are far from their true values. The proposed joint inversion is robust in the presence of the noise.

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