11th SAGA Biennial Technical Meeting and Exhibition
Swaziland, 16 - 18 September 2009, pages 55 - 58

Stochastic Modelling of the Velocity Structure: Beyond Joint Inversion Methods

K. Boomer[1], R. Brazier[2]

1. Bucknell University, USA, kb.boomer@bucknell.edu
2. Penn State Dubois, USA, rab27@psu.edu

ABSTRACT

The traditional approach to modelling seismic velocity structure of the Earth with multiple data sets (such as receiver functions and surface wave dispersion curves) is to perform a joint inversion. This method produced a single solution and little information on uncertainty in the models. We explore an alternative approach enhancing the multiple objective forward modelling method with a stochastic search and optimization. A genetic algorithm is well-suited to the velocity modelling setting as it supports parallel optimization, maintains diversity in the search space, and is more likely than random search procedures to converge to the global minima.

Key words: velocity modelling, stochastic optimization, genetic algorithm, joint inversion vs a forward model

INTRODUCTION

The joint inversion of surface wave dispersion curves and receiver functions has proved to be a useful tool in modelling the earth structure. The method proposed by Julià *et al.* (2000) uses the complementary nature of the surface wave dispersion (SWD) curves and receiver functions (RF) to provide constraints on the shear wave velocity, thus enabling a better modelling of the velocity structure than inverting either the SWD curve or RF separately. Both measures provide information about the crust and upper mantle. The SWD curves are an average velocity sampled over a range of depths, and their continuous nature provides smoothly changing gradients. RFs show peaks when a P-wave converts to an S-wave, which only occurs at sharp contrasts in velocity. The combined information describes the layers and gradients between them. This joint inversion method has been used extensively in a wide number of geographical regions and geologic complexities.

One of the challenges with the joint inversion approach is placing an error bound on the final velocity model, as noted in Julià *et al*. (2004). One proposal has been to apply a resampling approach such as a jackknife sample. However, in the joint inversion approach, a resampling approach will sample, with replacement, from the data provided (the observed receiver function and dispersion curve). This is problematic as a key component of, for example, the receiver function, may be omitted from such a resample. The resulting velocity model may thus miss a critical component of the geologic structure, or worse, not represent a viable geophysical model.

We propose a forward modelling approach combined with a stochastic search which will provide a measure of confidence with the final model.

THE MULTIPLE OBJECTIVE FORWARD MODELLING PROBLEM

The joint inversion approach iteratively calculates the velocity model given the data provided in the observed RF and the SWD curve. Each updated velocity model is used in turn to predict a RF and SWD curve, thus permitting (1) calculation of a point-wise misfit measure between the predicted and the observed RFs and SWD curves and (2) the calculation of an updated velocity model. Convergence occurs when the misfit function is minimized, which typically occurs within a small number of iterations (less than 20).

As opposed to iterative updating of a velocity model, the forward modelling approach selects a velocity model obtained via a search of the plausible velocity models (the model space) and calculates a predicted RF and SWD curve. The quality of the current velocity model is measured by the misfit between the predicted and observed RF and SWD curves, with the goal of minimizing the misfit. Each iteration of a forward model evaluates a new velocity model selected from the model space. A basic forward model uses a grid search to find the optimal model. However, such an exhaustive search is computationally prohibitive because of the large number of parameters estimated in a velocity model. There are many varied algorithms for searching the model space. Our interest lies in those algorithms that provide additional information beyond what is revealed in a joint inversion, such as a range of plausible models and an error bound on the estimates.

A forward model approach can be considered as a multiple objective problem in which one objective is the minimization of the misfit between observed and predicted RFs (f_1) and another objective is the minimization between the observed and predicted SWD curve (f_2). Additional objectives can easily be accommodated in a multiple objective problem.

Velocity models obtained from a probabilistic search mechanism are compared, based on the degree to which the misfit objective functions are minimized. The best solutions are those that minimize the misfit functions at least as well as the other solutions. If one solution is better than another, it is said to dominate that other solution. This can be visualized by plotting the two misfit objectives from multiple, varying velocity models (Figure 1).

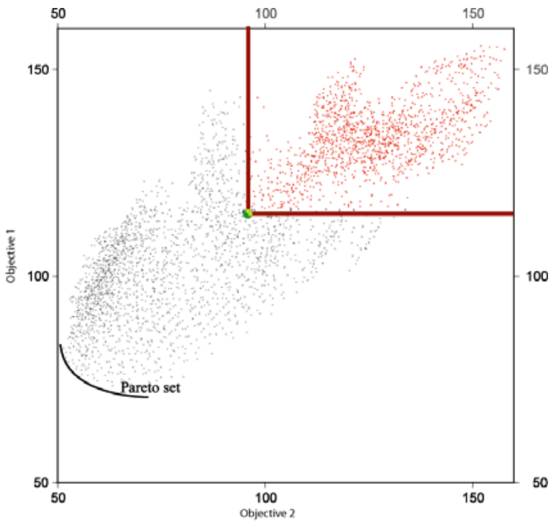

Figure 1: A single point on the graph can be seen to dominate over all the points with higher misfits in both directions (red). In other words, a point is considered dominant over another if both misfits are less than the others. The Pareto set is the set of nondominated solutions.

The set of points of interest are the points not dominated by any other points (i.e. the Pareto optimal set, Figure 1). All solutions in the Pareto optimal set are potentially as good as any of the other nondominated solutions. Thus, the multiple objective forward model approach provides multiple valid optimal solutions. From a statistical perspective, any one solution is one estimate of the true velocity structure, and multiple solutions can be used to provide an empirical confidence bound on the true velocity model.

STOCHASTIC OPTIMIZATION IN A MULTIPLE OBJECTIVE PROBLEM

In a stochastic optimization, a probabilistic mechanism is used to select velocity models from the model space. A random search (such as a random walk, simulated annealing, or a Monte Carlo Markov Chain) will search the space at random, and as a result, is prone to convergence at local minima. In comparison, importance sampling of the model space combines a probability component with a "learning" component to guide the search towards the global minimum. A genetic algorithm is one such search optimization algorithm.

Genetic Algorithms

The genetic algorithm (GA) is a stochastic search and optimization method based on the notion of evolution, in which initial solutions evolve toward the ideal solution (De Jong, *et al.*, 1997). The algorithm begins with an initial *population* of *individuals* (i.e. initial velocity models). At each *generation* (i.e. iteration), new individuals are obtained. In the terminology of evolution, the current individuals *mate* and *mutate* to form the next generation. That is, with some probability, two velocity models (now called *parents*) are selected and with a crossover probability p_c, two new individuals (called *children* or *offspring*) are obtained via proportional interpolation of the parent solutions, and with probability $1-p_c$, the two parent solutions enter into the next generation unchanged. Further, with probability p_m, the *children* or *offspring* mutate. In the case of velocity modelling, one layer may be changed by moving its current numerical value closer to the upper or lower bound of plausible values (provided by the user). The mating and mutation ensure a diverse population of velocity models is evaluated at each iteration, enhancing the efficiency of the search mechanism. Further, note that in a GA, multiple searches of the velocity model space are conducted in unison.

Deb *et al.*, 2002, expand the basic GA so that the probability of selecting an individual solution for mating is based on its degree of nondominance, with nondominating solutions being preferentially selected. In their Nondominated Sorting Genetic Algorithm II (NSGA-II), solutions are ranked based on their dominance in each generation (Figure 2).

The selection process for mating is based on a probability dependent on an individual's rank. Thus at each iteration there is a larger probability that a lower ranked individual will mate (survival of the fittest). The algorithm is considered converged when all individuals are consistently at rank=1 and thus the Pareto optimal set has been obtained.

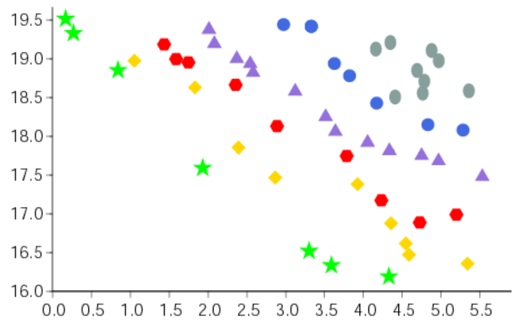

Figure 2: Ranking strategy. Green stars are the Pareto optimal set and have rank 1. If the green stars are removed, the yellow diamonds are optimal and have rank 2. Red hexagons have rank 3 etc.

DISCUSSION AND CONCLUSIONS

Multiple objective forward modelling of the velocity structure is particularly well suited to genetic algorithms as the objective function space tends to have several local maxima and minima. The parallel modelling of multiple velocity structures and the diverse nature of the individuals selected by the algorithm prevents the search algorithm from converging to local minima.

Genetic algorithms have been used in RF inversion problems (e.g. Sambridge and Kennett, 1996) and in SWD curve inversion problems (e.g. Mackenzie *et al.*, 2001). In addition, Dal Moro and Pipan (2007) used a generic Multi-Objective Evolutionary Algorithm (MOEA) to jointly invert SWD curves and reflection travel times. The NSGA II algorithm is a special case of a MOEA in which elitism is used in the selection process. This improves the computational efficiency, allowing for a larger population to be modelled. In addition, Deb *et al.* (2002) improved the MOEA mechanism for maintaining diversity to a crowded comparison approach to assess the density of the objective space. This means the user can avoid determining the appropriate distance sharing parameter.

Application

We have applied the NSGA II algorithm to the velocity model beneath the BOSA station on the Kaapvaal Craton. The velocity model obtained via a joint inversion (Kgaswane *et al.*, 2009) is used as a comparison.

Figure 3 shows the outcome of several runs, with varying crossover (p_c) and mutation (p_m) probabilities. Given that our objective is to minimize the two objective functions, we can see that mutation improves the Pareto optimal front, although this does increase the computational time. However, varying the crossover probability has a smaller impact. The improvement seen using mutation suggests the objective space does have local minima and that mutation enable the solutions (velocity models) to jump to other regions of the space, thus preventing convergence to local minima.

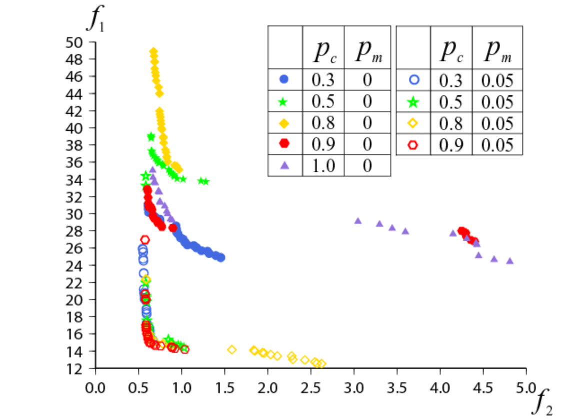

Figure 3: Pareto optimal sets for varying p_c and p_m probabilities. The objective functions are best minimized when mutation is included.

Based on this initial exploration applying the NSGA II algorithm, the objective functions f_1 and f_2 are best minimized with p_c=0.9 and p_m=0.05. Figure 4 plots the corresponding final velocity models in the Pareto optimal set. Overall, the predicted velocity models match realistic Earth models, with a lower velocity in the crust than in the upper mantle. However, individual layer fluctuations are too variable. This suggests a smoothing objective should be added (which can easily be accommodated in the NSGA II).

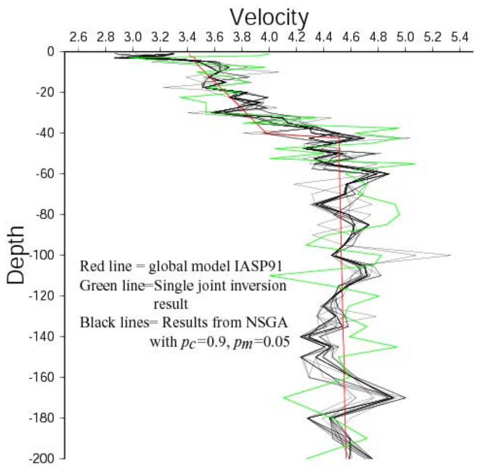

Figure 4: The predicted velocity models match realistic Earth models on a large scale, though large individual layer fluctuations suggests a need for a smoothing objective.

Finally, the misfit between the observed and predicted RF and the SWD curves show we are able to match the observed dispersion curve well, but smaller phases in the receiver function are not matched (Figure 5). This suggests modifying the objective function to include individual phases.

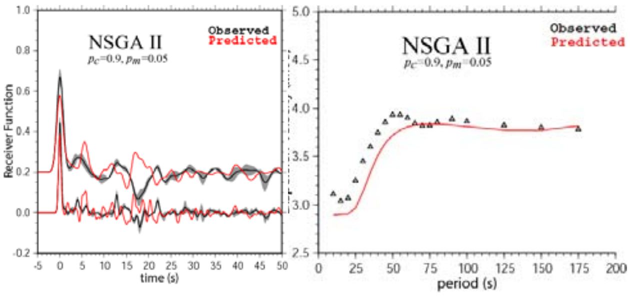

Figure 5: The misfits suggest overall encouraging results though modifications to the objective functions may be needed.

REFERENCES

Coello C., Lamont G., and Van Veldhuizen, D. (2007). *Evolutionary algorithms for solving multi-objective problems*, Springer, New York: 800 pp.

Dal Moro, G., and Pipan, M. (2007). Joint inversion of surface wave dispersion curves and reflection travel times via multi-objective evolutionary algorithms, *Journal of Applied Geophysics*, 61, pp. 56-81.

De Jong, K., Fogel, L., and Schwefel, H. (eds). (1997) *The Handbook of Evolutionary Computation*, Oxford University Press, Oxford: 997 pp.

Deb K., Agrawl, A., Pratap, A. and Meyarivan, T. (2002). A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II, *IEEE Transactions on Evolutionary Computation*, 6, pp. 182-197.

Julià J., Ammon, C., Herrmann, R., and Corrieg, A. (2000). Joint inversion of receiver functions and surface wave dispersion observations, *Geophysical Journal International*, 143, pp. 99-112.

Kgaswane, E, Nyblade, A.A, Julià, J., Dirks, P.H.G.M., Durrheim, R.J. and Pasyanos, M. (2009). Shear wave velocity structure of the lower crust in Southern Africa: Evidence for compositional heterogeneity within Archaean and Proterozoic terrains, *Journal of Geophysical Research*, in press.

Mackenzie G., Maguire, P., Denton, P., Morgan, J. and Warner, M. (2001). Shallow seismic velocity structure of the Chicxulub impact crater from modelling of Rg dispersion using a genetic algorithm, *Tectonophysics*, 338, pp. 97-112.

Sambridge M., and Kennett, B. (1996). Genetic algorithm inversion for receiver functions with application to crust and uppermost mantle structure beneath Eastern Australia, *Geophysical Research Letters,* 23(14), pp. 1829-1832.

Sandvol E., Seber. D.,. Barazangri., M., Vernon, F., Mellors, R.,. Al-Amri, A. (1998). Lithospheric seismic velocity discontinuities beneath the Arabian Shield, *Geophysical Research Letters*, 25(15), pp. 2873-2876.